

**Errata and Supplements to  
Discrete Convex Analysis (SIAM, 2003)  
2nd printing (soft cover), 2013, ISBN 978-1-611972-55-9**

- Page 77, line 2 from bottom:  $+\infty \leq +\infty \implies +\infty \geq +\infty$
- Page 94, line 4 from bottom: Delete “is as follows.”
- Page 113, line 12 from bottom (last paragraph of the proof of Theorem 4.18):  
Let  $p$  be such that the optimal solutions to (A) with respect to  $p$  form a minimal face of the feasible region of (A).
- Page 117, line 1 (below Figure 4.1):  
 $Q \subseteq \mathbf{Z}^V$  is an  $M^{\sharp}$ -convex set  $\implies Q \subseteq \mathbf{Z}^V$  is said to be an  $M^{\sharp}$ -convex set
- Page 139, Proposition 6.8 (2): Condition (6.26) follows from (6.27), and hence (6.26) is redundant.
- Page 143, three lines above Theorem 6.13:  
 $f_{[a,b]}$  for an integer interval  $[a, b]$   $\implies$   
 $f_{[a,b]}$  for the restriction of  $f$  to an integer interval  $[a, b]$
- Page 143, Theorem 6.13 (8) [Convolution of M-convex functions]  
The proof here makes use of transformation by a network, but an alternative direct proof can be found in:  
K. Murota: On infimal convolution of M-convex functions, RIMS Kokyuroku, No.1371 (2004), 20–26.
- Page 151, Theorem 6.30, Proof of Claim 1:  
The final step reads: “This shows (B-EXC<sub>+</sub>[ $\mathbf{R}$ ]) for  $\bar{B}$ . Therefore,  $B$  is an M-convex set.” Before we can argue in this way, we have to verify  $\bar{B} \cap \mathbf{Z}^V = B$ , which is possible.
- Page 152, Section 6.8: A characterization of gross substitutes property in terms of an exchange property is also found in:  
H. Reijnierse, A. van Gallekom, and J. A. M. Potters: Verifying gross substitutability, *Economic Theory*, **20** (2002), 767–776.
- Page 172, Proof of Theorem 6.74:  
“Theorem 6.4 can be strengthened to a statement that (M-EXC[ $\mathbf{Z}$ ]) and (M-EXC<sub>loc</sub>[ $\mathbf{Z}$ ]) are equivalent if  $\text{dom } f$  satisfies (Q-EXC<sub>w</sub>). (This can be shown by modifying the proof of Claim 2 in the proof of Theorem 6.4.)”  
The detail of the argument can be found in a memorandum of A. Shioura: Level set characterization of M-convex functions (February 1998); see Claim 2 on page 6.

- Page 185, Theorem 7.14 [L-optimality criterion]  
The proof here makes use of the optimality criterion for integrally convex functions, but an alternative direct proof can be found in:  
K: Murota: A proof of the L-optimality criterion theorem, unpublished note, July 2004,  
<https://kzmurota.fpark.tmu.ac.jp/paper/loptimality04.pdf>
- Page 186, Theorem 7.17  
“with a bounded nonempty effective domain”  
 $\implies$  “that is convex-extensible or has a bounded nonempty effective domain”  
For an L-convex function  $g$ , the boundedness assumption on the effective domain is intended to mean the boundedness of  $\text{dom } g$  intersected with a coordinate plane  $\{p \mid p(v) = 0\}$  for some  $v \in V$ . Note that the effective domain of an L-convex function has the invariance in the direction of  $\mathbf{1}$ .
- Page 219, Theorem 8.17 [M-convex intersection theorem]  
The proof here makes use of the M-separation theorem, but an alternative direct proof can be found in:  
K. Murota: A proof of the M-convex intersection theorem, RIMS Kokyuroku, No.1371 (2004), 13–19.
- Page 228, line 5, Theorem 8.33:  $f_2(x + \chi_{u_{i+1}} - \chi_{v_i}) \implies f_2(x - \chi_{u_{i+1}} + \chi_{v_i})$
- Page 228, line 13, Theorem 8.34:  
 $f_2(x^\alpha + \alpha(\chi_{u_{i+1}} - \chi_{v_i})) \implies f_2(x^\alpha - \alpha(\chi_{u_{i+1}} - \chi_{v_i}))$
- Page 231, Proof of Theorem 8.42:  
There was a confusion between  $L_2$ -convexity and  $L_2^{\natural}$ -convexity. Please change the first paragraph of the proof (“It suffices to consider ..... found in Murota–Shioura [153].”) to the following text:  
  
To emphasize the essence we give a proof for the integral convexity of an  $L_2$ -convex set, from which the integral convexity of an  $L_2^{\natural}$ -convex set follows. This implies, by Theorem 3.29, the integral convexity of an  $L_2^{\natural}$ -convex function  $g$  with a bounded effective domain, since  $\arg \min g[-x]$  is an  $L_2^{\natural}$ -convex set for any  $x \in \mathbf{R}^V$  by Proposition 8.40. A complete proof can be found in Murota–Shioura [153].  
  
Integral convexity of an  $L_2^{\natural}$ -convex set ( $L_2$ -convex set) also follows from its box-total dual integrality (see Theorem 5.1 of S. Moriguchi and K. Murota: Note on the polyhedral description of the Minkowski sum of two L-convex sets, Japan Journal of Industrial and Applied Mathematics, Vol.40 (2023), No.1, 223–263. <https://doi.org/10.1007/s13160-022-00512-3> (Open access))
- Page 305, Section 10.3.1:  
A detailed analysis of the steepest descent algorithm for L-convex functions can be found in:

K. Murota and A. Shioura: Exact bounds for steepest descent algorithms of L-convex function minimization, *Operations Research Letters*, **42** (2014), 361–366.

- Page 331, ( $-M^{\natural}$ -SWG[S] $[Z]$ ): For  $x \in \arg \min U[-p] \implies$  For  $x \in \arg \max U[-p]$
- Page 333, “(SNC) for  $\tilde{U} \implies M^{\natural}$ -concavity for  $U$ ”:  
The converse of this statement is also true, that is,

$$(\text{SNC}) \text{ for } \tilde{U} \iff M^{\natural}\text{-concavity for } U$$

holds. See

K. Murota: Multiple exchange property for  $M^{\natural}$ -concave functions and valuated matroids, *Mathematics of Operations Research*, **43** (2018), 7810–788.

<https://doi.org/10.1287/moor.2017.0882> , arXiv: <http://arxiv.org/abs/1608.07021>

K. Murota: A stronger multiple exchange property for  $M^{\natural}$ -concave functions, *Japan Journal of Industrial and Applied Mathematics*, **35** (2018), 411–421.

<https://doi.org/10.1007/s13160-017-0278-4> , arXiv: <http://arxiv.org/abs/1706.09222>

- Page 365, [39]: P. G. Doyle and J. L. Snell: *Random Walks and Electrical Networks*, Mathematical Association of America, Washington DC, 1984.
- Page 369, [105] (S. Iwata and M. Shigeno): (2003)  $\implies$  (2002)
- Page 372, [143] (K. Murota): (1999)  $\implies$  (1998)
- Page 373, [153] (K. Murota and A. Shioura)  
and by Favati–Tardella  $\implies$  and Favati–Tardella
- Page 375, [192] (A. Shioura): (2003)  $\implies$  (2004)
- Page 376, [202]: D. M. Topkis,  $\implies$  D. M. Topkis:

#### Updates of bib-infor:

- [33] V. Danilov, G. Koshevoy, and C. Lang: Gross substitution, discrete convexity, and submodularity, *Discrete Applied Mathematics*, **131** (2003), 283–298.
- [47] A. Eguchi and S. Fujishige: An extension of the Gale–Shapley stable matching algorithm to a pair of  $M^{\natural}$ -concave functions, *Discrete Mathematics and Systems Science Research Report*, No. 02-05, Division of Systems Science, Osaka University, November 2002. (This is the final form; no journal paper exists.)
- [69] S. Fujishige and Z. Yang: A note on Kelso and Crawford’s gross substitutes condition, *Mathematics of Operations Research*, **28** (2003), 463–469.

(end)